Student: _____	**Instructor:** Alfredo Alvarez	**Assignment:** math 131437 Free Response with Help 51
Date: _____	**Course:** Math 1314 Summer 2018	

1. Solve the equation by factoring.

$9x^2 + 21x - 8 = 0$

The solution set is { }.
(Use a comma to separate answers as needed.)

2. Use factoring to solve the quadratic equation. Check by substitution or by using a graphing utility and identifying x-intercepts.

$2x^2 = 9x + 18$

The solution set is { }.
(Use commas to separate answers as needed. Type repeated roots only once.)

3. Solve for x using the quadratic formula.

$x^2 - 8x + 20 = 0$

The solution set is { }.
(Type an exact answer, using radicals as needed. Express complex numbers in terms of i. Use a comma to separate answers as needed.)

4. Solve the following equation.

$2x^2 - 20x + 50 = 0$

The solution set is { }.
(Use a comma to separate answers as needed.)

5. Solve the given radical equation. Check all proposed solutions.

$\sqrt{3x + 25} = x + 7$

Select the correct choice below and, if necessary, fill in the answer box to complete your choice.

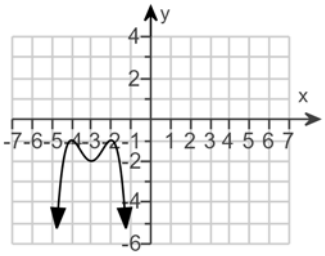
- **A.** The solution set is { }.
  (Use a comma to separate answers as needed.)
- **B.** There is no solution.

6. Use the graph to determine

(a) open intervals on which the function is increasing, if any.

(b) open intervals on which the function is decreasing, if any.

(c) open intervals on which the function is constant, if any.



(a) Select the correct choice below and, if necessary, fill in the answer box to complete your choice.

- **A.** The function is increasing on the interval(s) _____.
  (Type your answer in interval notation. Use a comma to separate answers as needed.)
- **B.** The function is never increasing.

(b) Select the correct choice below and, if necessary, fill in the answer box to complete your choice.

- **A.** The function is decreasing on the interval(s) _____.
  (Type your answer in interval notation. Use a comma to separate answers as needed.)
- **B.** The function is never decreasing.

(c) Select the correct choice below and, if necessary, fill in the answer box to complete your choice.

- **A.** The function is constant on the interval(s) _____.
  (Type your answer in interval notation. Use a comma to separate answers as needed.)
- **B.** The function is never constant.

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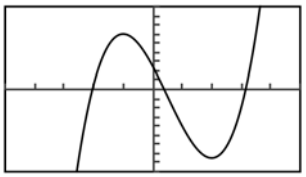
7. The graph and equation of the function f are given.
a. Use the graph to find any values at which f has a relative maximum, and use the equation to calculate the relative maximum for each value.
b. Use the graph to find any values at which f has a relative minimum, and use the equation to calculate the relative minimum for each value.

$f(x) = 2x^3 - 3x^2 - 12x + 5$



$[-5,5,1]$ by $[-18,18,2]$

a. Select the correct choice below and, if necessary, fill in the answer boxes to complete your choice.

- **A.** The function f has (a) relative maxima(maximum) at _____ and the relative maxima(maximum) are(is) _____.
  (Use a comma to separate answers as needed.)
- **B.** The function f has no relative maxima.

b. Select the correct choice below and, if necessary, fill in the answer boxes to complete your choice.

- **A.** The function f has (a) relative minima(minimum) at _____ and the relative minima(minimum) are(is) _____.
  (Use a comma to separate answers as needed.)
- **B.** The function f has no relative minima.

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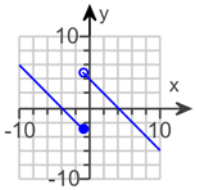
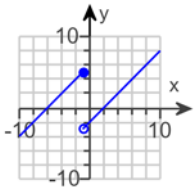
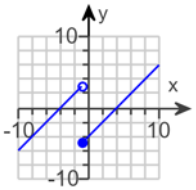
8. The domain of the piecewise function is $(-\infty,\infty)$.
a. Graph the function.
b. Use your graph to determine the function's range.

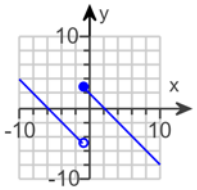
$$f(x)=\begin{cases} x+4 & \text{if } x<-1 \\ x-4 & \text{if } x\geq -1 \end{cases}$$

a. Choose the correct graph below.

A.


B.


C.


D.


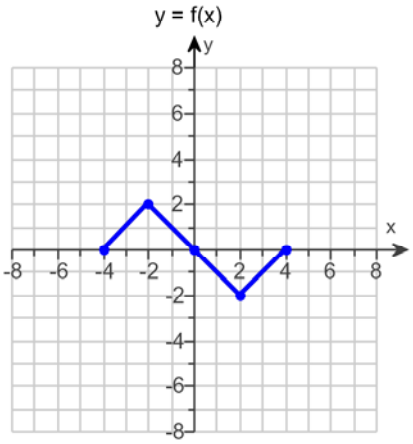
b. The range of f(x) is _____. (Type your answer in interval notation.)

9. Find the difference quotient of f; that is, find $\frac{f(x+h)-f(x)}{h}$, $h\neq 0$, for the following function. Be sure to simplify.

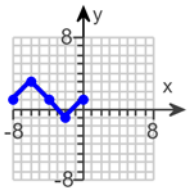
$f(x)=x^2-7x+6$

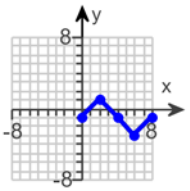
$\frac{f(x+h)-f(x)}{h}=$ _____ (Simplify your answer.)

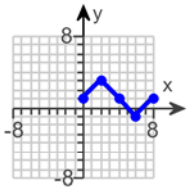
10. Use the graph of $y=f(x)$ to graph the function $g(x)=f(x+4)+1$.

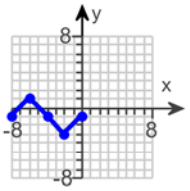


Choose the correct graph of g below.

A.


B.


C.


D.


11. Find the domain of the function.

$f(x)=\sqrt{28-4x}$

What is the domain of f?

_____ (Type your answer in interval notation.)

12. First find $f+g$, $f-g$, fg and $\frac{f}{g}$. Then determine the domain for each function.

$f(x)=3x^2+6x-9,\ g(x)=x-1$

$(f+g)(x)=$ ☐ (Simplify your answer.)

What is the domain of $f+g$?

- ○ $[0,\infty)$
- ○ $\left(\frac{10}{7},\infty\right)$
- ○ $\left(-\infty,\frac{10}{7}\right)\cup\left(\frac{10}{7},\infty\right)$
- ○ $(-\infty,\infty)$

$(f-g)(x)=$ ☐ (Simplify your answer.)

What is the domain of $f-g$?

- ○ $\left(\frac{10}{7},\infty\right)$
- ○ $\left(-\infty,\frac{3}{2}\right)\cup\left(\frac{3}{2},\infty\right)$
- ○ $(-\infty,\infty)$
- ○ $[0,\infty)$

$(fg)(x)=$ ☐

What is the domain of fg?

- ○ $\left(\frac{8}{5},\infty\right)$
- ○ $(-\infty,\infty)$
- ○ $\left(-\infty,\frac{8}{5}\right)\cup\left(\frac{8}{5},\infty\right)$
- ○ $(-\infty,1)\cup(1,\infty)$

$\left(\frac{f}{g}\right)(x)=$ ☐ (Simplify your answer.)

What is the domain of $\frac{f}{g}$?

- ○ $(1,\infty)$
- ○ $[0,\infty)$
- ○ $(-\infty,1)\cup(1,\infty)$
- ○ $(-\infty,\infty)$

13. For $f(x) = 4 - x$ and $g(x) = 2x^2 + x + 5$, find the following functions.

a. $(f \circ g)(x)$; **b.** $(g \circ f)(x)$; **c.** $(f \circ g)(2)$; **d.** $(g \circ f)(2)$

a. $(f \circ g)(x) =$ ☐
(Simplify your answer.)

b. $(g \circ f)(x) =$ ☐
(Simplify your answer.)

c. $(f \circ g)(2) =$ ☐

d. $(g \circ f)(2) =$ ☐

14. Find the distance between the pair of points.

(4,7) and (13,19)

The distance between the points is ☐ units.
(Round to two decimal places as needed.)

15. Find the midpoint of the line segment with the given endpoints.

(6,4) and (8,10)

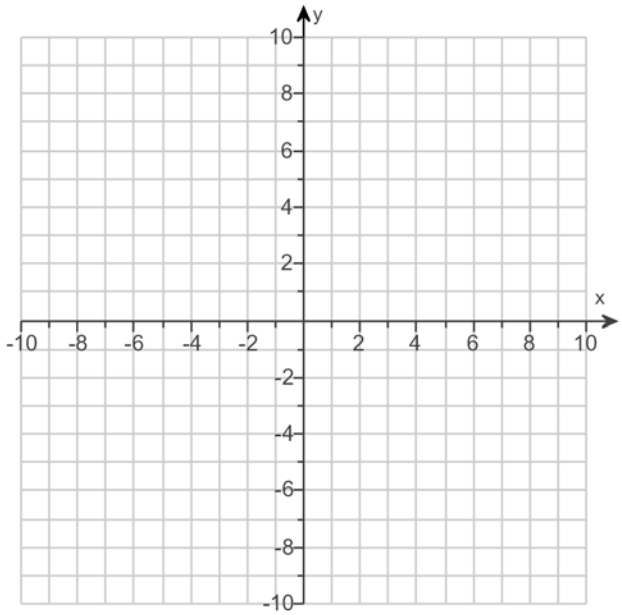
The midpoint of the segment is ☐.
(Type an ordered pair.)

16. Complete the square and write the equation of the circle in standard form. Then determine the center and radius of the circle to graph the equation.

$x^2 + y^2 + 10x + 8y + 37 = 0$

The equation in standard form is ☐.
(Simplify your answer.)

Use the graphing tool to graph the circle.



17. In the following exercise, find the coordinates of the vertex for the parabola defined by the given quadratic function.

$f(x) = 3x^2 + 6x + 1$

The vertex is ☐. (Type an ordered pair.)

18. Find the coordinates of the vertex for the parabola defined by the given quadratic function.

$f(x) = -x^2 - 2x + 10$

The vertex is [ ]. (Type an ordered pair.)

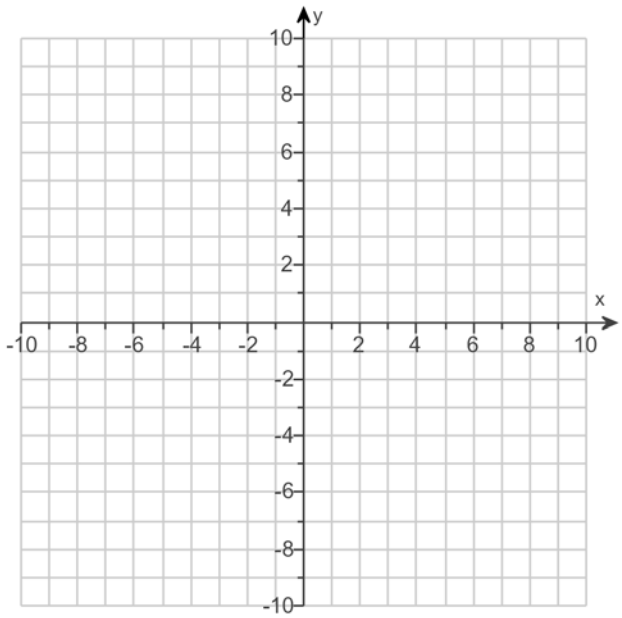
19.

Use the vertex and intercepts to sketch the graph of the quadratic function. Give the equation of the parabola's axis of symmetry. Use the graph to determine the domain and range of the function.

$f(x) = (x + 2)^2 - 9$

Use the graphing tool to graph the function. Use the vertex and one of the intercepts when drawing the graph.

The axis of symmetry is [ ].
(Type an equation. Simplify your answer.)
The domain of the function is [ ].
(Type your answer in interval notation.)
The range of the function is [ ].
(Type your answer in interval notation.)



20.

Use the vertex and intercepts to sketch the graph of the quadratic function. Give the equation for the parabola's axis of symmetry. Use the parabola to identify the function's domain and range.

$f(x) = (x - 1)^2 + 2$

Use the graphing tool to graph the equation. Use the vertex and the y-intercept when drawing the graph.

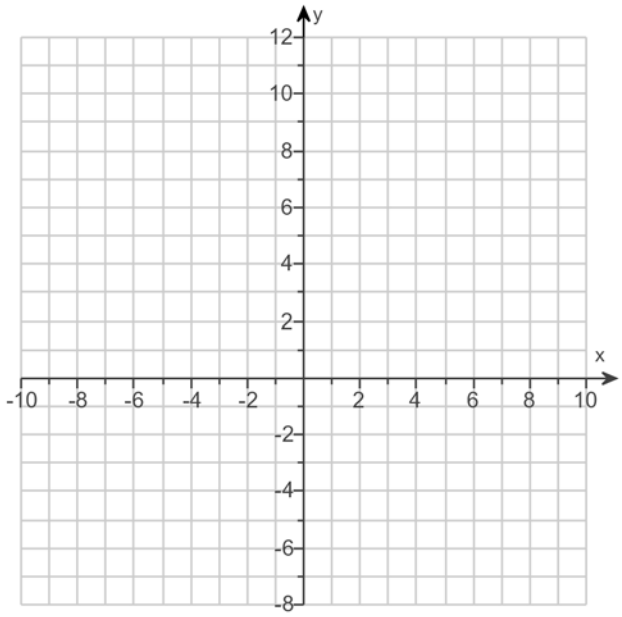
The axis of symmetry is [ ].
(Simplify your answer. Type an equation.)

Identify the function's domain.

The domain is [ ].
(Type the answer in interval notation.)

Identify the function's range.

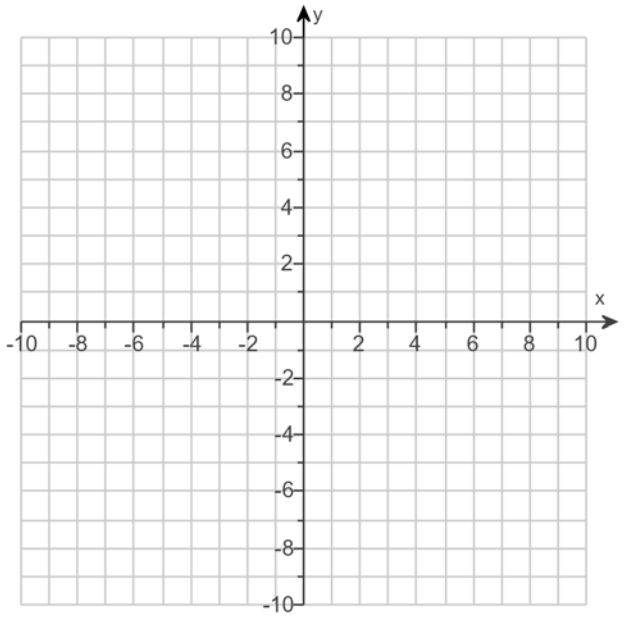
The range is [ ].
(Type the answer in interval notation.)



21.

Use the vertex and intercepts to sketch the graph of the quadratic function. Give the equation of the parabola's axis of symmetry. Use the graph to determine the function's domain and range.

$f(x) = x^2 + 4x + 3$



Use the graphing tool to graph the equation. Use the vertex and one of the intercepts when drawing the graph.

The axis of symmetry is [ ].
(Type an equation.)

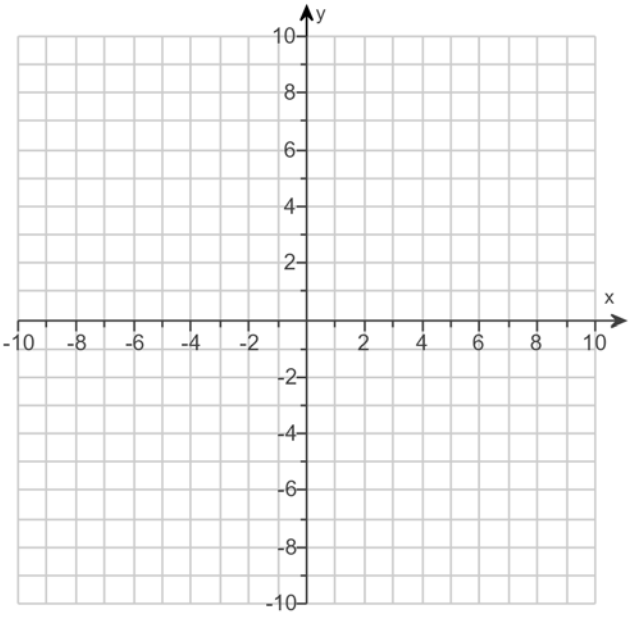
The domain of f is [ ].
(Type your answer in interval notation.)

The range of f is [ ].
(Type your answer in interval notation.)

22.

Use the vertex and intercepts to sketch the graph of the quadratic function. Give the equation of the parabola's axis of symmetry. Use the graph to determine the domain and range of the function.

$f(x) = 2x - x^2 + 3$



Use the graphing tool to graph the equation. Use the vertex and one of the intercepts to draw the graph.

The axis of symmetry is [ ].
(Type an equation.)

The domain of the function is [ ].
(Type your answer in interval notation.)

The range of the function is [ ].
(Type your answer in interval notation.)

23. Solve the equation $x^3 + 2x^2 - 5x - 6 = 0$ given that 2 is a zero of $f(x) = x^3 + 2x^2 - 5x - 6$.

The solution set is { [ ] }. (Use a comma to separate answers as needed.)

24. The following function is given.

$f(x) = 7x^3 - 5x^2 - 63x + 45$

a. List all rational zeros that are possible according to the Rational Zero Theorem. Choose the correct answer below.

- **A.** $\pm 1, \pm 7, \pm \frac{1}{3}, \pm \frac{7}{3}, \pm \frac{1}{9}, \pm \frac{7}{9}, \pm \frac{1}{5}, \pm \frac{7}{5}, \pm \frac{1}{15}, \pm \frac{7}{15}, \pm \frac{1}{45}, \pm \frac{7}{45}$
- **B.** $\pm 1, \pm 7, \pm \frac{1}{3}, \pm \frac{7}{3}, \pm \frac{1}{6}, \pm \frac{7}{6}, \pm \frac{1}{5}, \pm \frac{7}{5}, \pm \frac{1}{15}, \pm \frac{7}{15}, \pm \frac{1}{45}, \pm \frac{7}{45}$
- **C.** $\pm 1, \pm 3, \pm 9, \pm 5, \pm 15, \pm 45, \pm \frac{1}{7}, \pm \frac{3}{7}, \pm \frac{9}{7}, \pm \frac{5}{7}, \pm \frac{15}{7}, \pm \frac{45}{7}$
- **D.** $\pm 1, \pm 3, \pm 6, \pm 5, \pm 15, \pm 45, \pm \frac{1}{7}, \pm \frac{3}{7}, \pm \frac{6}{7}, \pm \frac{5}{7}, \pm \frac{15}{7}, \pm \frac{45}{7}$

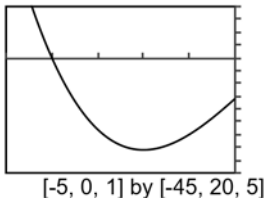
b. Use synthetic division to test several possible rational zeros in order to identify one actual zero.

One rational zero of the given function is ☐.
(Simplify your answer.)

c. Use the zero from part **(b)** to find all the zeros of the polynomial function.

The zeros of the function $f(x) = 7x^3 - 5x^2 - 63x + 45$ are ☐.
(Simplify your answer. Type an integer or a fraction. Use a comma to separate answers as needed.)

25. An incomplete graph of the polynomial function $f(x) = -x^3 + x^2 + 16x - 16$ is shown on the right.



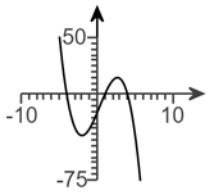
a. Find all zeros of the function.

b. Without using a graphing utility, draw a complete graph of the function.

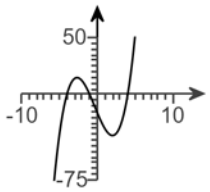
a. The zeros are ☐.
(Use a comma to separate answers as needed.)

b. Choose the correct graph of the function below. The scale for each graph is $[-10, 10, 1]$ by $[-75, 50, 5]$.

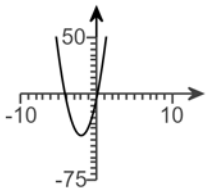
- **A.**



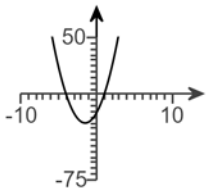
- **B.**



- **C.**



- **D.**



26. Fill in the blank so that the resulting statement is true.

Based on the synthetic division shown below, the equation of the slant asymptote of $f(x) = \frac{2x^2 - 7x + 3}{x - 6}$ is _____.

$$\begin{array}{r|rrr} 6 & 2 & -7 & 3 \\ & & 12 & 30 \\ \hline & 2 & 5 & 33 \end{array}$$

Based on the synthetic division shown, the equation of the slant asymptote of $f(x) = \frac{2x^2 - 7x + 3}{x - 6}$ is [ ]. (Type an equation.)

27. Find the vertical asymptotes, if any, and the values of x corresponding to holes, if any, of the graph of the rational function.

$$f(x) = \frac{x}{x - 3}$$

Select the correct choice below and, if necessary, fill in the answer box to complete your choice. (Type an equation. Use commas to separate answers as needed.)

- **A.** The vertical asymptote(s) is(are) _____ and hole(s) corresponding to _____.
- **B.** The vertical asymptote(s) is(are) _____. There are no holes.
- **C.** There are no vertical asymptotes but there is(are) hole(s) corresponding to _____.
- **D.** There are no discontinuities.

28. Find the vertical asymptotes, if any, and the values of x corresponding to holes, if any, of the graph of the rational function.

$$f(x) = \frac{x - 6}{x^2 - 11x + 30}$$

Select the correct choice below and, if necessary, fill in the answer box(es) to complete your choice. (Type an integer or a fraction. Use a comma to separate answers as needed.)

- **A.** Hole(s) at x = _____
- **B.** Vertical asymptote(s) at x = _____ and hole(s) at x = _____
- **C.** Vertical asymptote(s) at x = _____
- **D.** There are no discontinuities.

29. Find the horizontal asymptote, if any, of the graph of the rational function.

$$f(x) = \frac{11x}{3x^2 + 7}$$

Select the correct choice below and, if necessary, fill in the answer box to complete your choice.

- **A.** The horizontal asymptote is _____. (Type an equation.)
- **B.** There is no horizontal asymptote.

30. Find the horizontal asymptote, if any, of the graph of the rational function.

$$g(x) = \frac{28x^2}{7x^2 + 8}$$

Select the correct choice below and, if necessary, fill in the answer box to complete your choice.

- **A.** The horizontal asymptote is _____. (Type an equation.)
- **B.** There is no horizontal asymptote.

31. Find the domain of the logarithmic function.

$$f(x) = \log(13 - x)$$

The domain of $f(x) = \log(13 - x)$ is [ ].
(Type your answer in interval notation.)

32. Use properties of logarithms to expand the logarithmic expression as much as possible. Evaluate logarithmic expressions without using a calculator if possible.

$$\log_b\left(\frac{x^2 y}{z^3}\right)$$

$$\log_b\left(\frac{x^2 y}{z^3}\right) = \square$$

33. Use properties of logarithms to expand the logarithmic expression as much as possible. Evaluate logarithmic expressions without using a calculator if possible.

$$\ln\left[\frac{x^7\sqrt{x^2+4}}{(x+4)^5}\right]$$

$$\ln\left[\frac{x^7\sqrt{x^2+4}}{(x+4)^5}\right] = \square$$

34. Solve the following exponential equation by expressing each side as a power of the same base and then equating exponents.

$$27^{x+1} = 243^{x-7}$$

The solution set is $\{\square\}$.

35. Solve the following exponential equation by taking the natural logarithm on both sides. Express the solution in terms of natural logarithms. Then, use a calculator to obtain a decimal approximation for the solution.

$9e^{5x} = 1251$

What is the solution in terms of natural logarithms?

The solution set is { }.
(Use a comma to separate answers as needed. Simplify your answer. Use integers or fractions for any numbers in the expression.)

What is the decimal approximation for the solution?

The solution set is { }.
(Use a comma to separate answers as needed. Round to two decimal places as needed.)

36. Solve the exponential equation. Express the solution in terms of natural logarithms. Then use a calculator to obtain a decimal approximation for the solution.

$3^{(x+2)} = 469$

What is the solution in terms of natural logarithms?

The solution set is { }.
(Use a comma to separate answers as needed. Simplify your answer. Use integers or fractions for any numbers in the expression.)

What is the decimal approximation for the solution?

The solution set is { }.
(Use a comma to separate answers as needed. Round to two decimal places as needed.)

37. Solve the following logarithmic equation. Be sure to reject any value of x that is not in the domain of the original logarithmic expression. Give the exact answer.

$\log_2(x+20) = 2$

Select the correct choice below and, if necessary, fill in the answer box to complete your choice.

- **A.** The solution set is { }. (Type an integer or a simplified fraction.)
- **B.** There is no solution.

38. Solve the logarithmic equation. Be sure to reject any value of x that is not in the domain of the original logarithmic expressions. Give an exact answer.

$\log_8 x + \log_8(7x-1) = 1$

Select the correct choice below and, if necessary, fill in the answer box to complete your choice.

- **A.** The solution set is { }. (Type an exact answer in simplified form.)
- **B.** There is no solution.

39. Solve the logarithmic equation. Be sure to reject any value of x that is not in the domain of the original logarithmic expressions. Give the exact answer.

$$\log_5(x+1) + \log_5(x+121) = 4$$

Select the correct choice below and, if necessary, fill in the answer box to complete your choice.

- **A.** The solution set is $\{\ \square\ \}$.
  (Simplify your answer. Use a comma to separate answers as needed.)
- **B.** There is no solution.

40. Solve the logarithmic equation. Be sure to reject any value of x that is not in the domain of the original logarithmic expressions. Give the exact answer.

$$\log_4(x+11) - \log_4(x-4) = 2$$

Select the correct choice below and, if necessary, fill in the answer box to complete your choice.

- **A.** The solution set is $\{\ \square\ \}$.
  (Simplify your answer. Use a comma to separate answers as needed.)
- **B.** There is no solution.

41. Solve the logarithmic equation. Be sure to reject any value of x that is not in the domain of the original logarithmic expressions. Give the exact answer.

$$\log x + \log(x+2) = \log 63$$

Select the correct choice below and, if necessary, fill in the answer box to complete your choice.

- **A.** The solution set is $\{\ \square\ \}$.
  (Simplify your answer. Use a comma to separate answers as needed.)
- **B.** There is no solution.

42. The formula $A = 15.7\,e^{0.0409t}$ models the population of a US state, A, in millions, t years after 2000.

a. What was the population of the state in 2000?
b. When will the population of the state reach 18.7 million?

a. In 2000, the population of the state was $\square$ million.

b. The population of the state will reach 18.7 million in the year $\square$.
(Round down to the nearest year.)

43. Complete the table for a savings account subject to 2 compoundings yearly.

$$\left[A = P\left(1 + \frac{r}{n}\right)^{nt}\right]$$

Amounted Invested	Number of Compounding Periods	Annual Interest Rate	Accumulated Amount	Time t in Years
$14,500	2	4.5%	$23,000	?

Let A represent the accumulated amount, P the amount invested, n the number of compounding periods, r the annual interest rate, and t the time. Find the time, t.

t = ☐ years
(Do not round until the final answer. Then round to one decimal place as needed.)

---

44. An artifact originally had 16 grams of carbon-14 present. The decay model $A = 16\,e^{-0.000121t}$ describes the amount of carbon-14 present after t years. Use the model to determine how many grams of carbon-14 will be present in 9692 years.

The amount of carbon-14 present in 9692 years will be approximately ☐ grams.
(Round to the nearest whole number.)

---

45. Prehistoric cave paintings were discovered in a cave in France. The paint contained 29% of the original carbon-14. Use the exponential decay model for carbon-14, $A = A_0\,e^{-0.000121t}$, to estimate the age of the paintings.

The paintings are approximately ☐ years old. (Round to the nearest integer.)

---

46. Use the formula $t = \frac{\ln 2}{k}$ that gives the time for a population, with a growth rate k, to double, to answer the following questions.

The growth model $A = 3\,e^{0.003t}$ describes the population, A, of a country in millions, t years after 2003.

a. What is the country's growth rate?

☐ %

b. How long will it take the country to double its population?

☐ years (Round to the nearest whole number.)

---

47. Solve the given system of equations.

$$\begin{aligned} x + y + 8z &= -22 \\ x + y + 4z &= -10 \\ x - 7y + 7z &= -11 \end{aligned}$$

Select the correct choice below and fill in any answer boxes within your choice.

- **A.** There is one solution. The solution set is $\{($ ☐ , ☐ , ☐ $)\}$. (Simplify your answers.)
- **B.** There are infinitely many solutions.
- **C.** There is no solution.

---

48. Write the first four terms of the sequence whose general term is given.

$$a_n = \frac{2n}{n+5}$$

$a_1 =$ ☐ (Simplify your answer.)

$a_2 =$ ☐ (Simplify your answer.)

$a_3 =$ ☐ (Simplify your answer.)

$a_4 =$ ☐ (Simplify your answer.)

---

49. Find the indicated sum. $\sum_{i=1}^{4} i(i+1)$

$\sum_{i=1}^{4} i(i+1) =$ ☐ (Simplify your answer.)

---

50. Use the binomial theorem to expand the binomial.

$$(3x-2)^3$$

$(3x-2)^3 =$ ☐ (Simplify your answer.)

---

51. Write the first three terms of the binomial expansion, expressing the result in simplified form.

$$(x+7)^6$$

The first three terms of the binomial expansion are ☐.
(Simplify your answer.)

1. $\frac{1}{3}, -\frac{8}{3}$

---

2. $-\frac{3}{2}, 6$

---

3. $4+2i, 4-2i$

---

4. 5

---

5. A. The solution set is $\{$ **−3** $\}$.(Use a comma to separate answers as needed.)

---

6. A. The function is increasing on the interval(s) **$(-\infty, -4), (-3, -2)$**.
(Type your answer in interval notation. Use a comma to separate answers as needed.)

A. The function is decreasing on the interval(s) **$(-4, -3), (-2, \infty)$**.
(Type your answer in interval notation. Use a comma to separate answers as needed.)

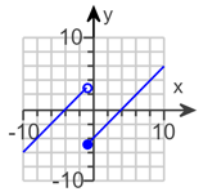
B. The function is never constant.

---

7. A.
The function f has (a) relative maxima(maximum) at **−1** and the relative maxima(maximum) are(is) **12**.
(Use a comma to separate answers as needed.)

A.
The function f has (a) relative minima(minimum) at **2** and the relative minima(minimum) are(is) **−15**.
(Use a comma to separate answers as needed.)

---

8. 

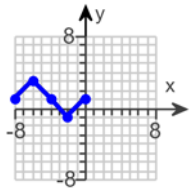
C.

$(-\infty, \infty)$

---

9. $2x + h - 7$

---

10. 

A.

---

11. $(-\infty,7]$

---

12. $3x^2+7x-10$

$(-\infty,\infty)$

$3x^2+5x-8$

$(-\infty,\infty)$

$3x^3+3x^2-15x+9$

$(-\infty,\infty)$

$3x+9$

$(-\infty,1)\cup(1,\infty)$

---

13. $-2x^2-x-1$

$2x^2-17x+41$

-11

15

---

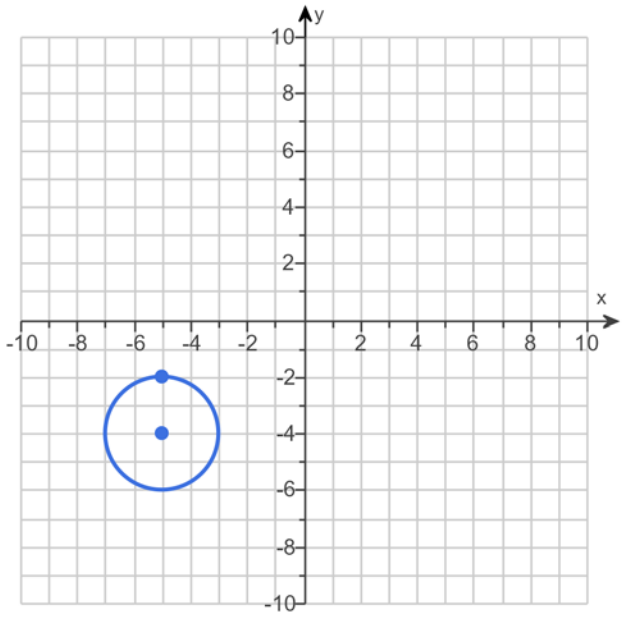
14. 15

---

15. (7,7)

---

16. $(x+5)^2+(y+4)^2=4$



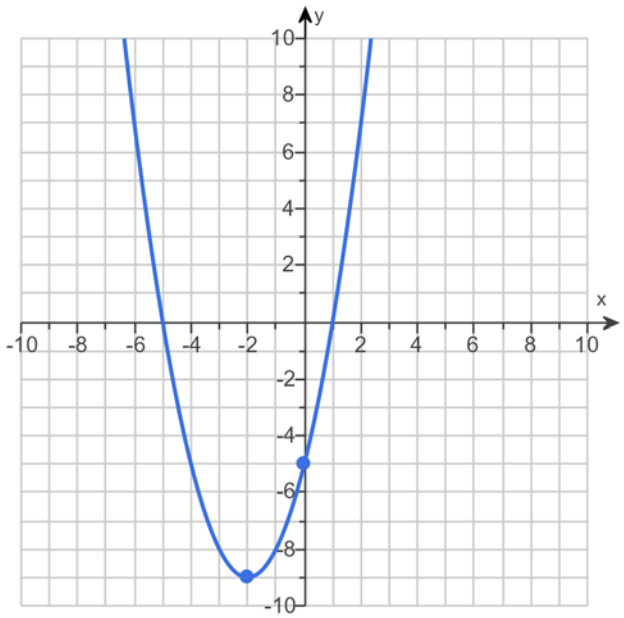
---

17. $(-1,-2)$

---

18. $(-1,11)$

19.

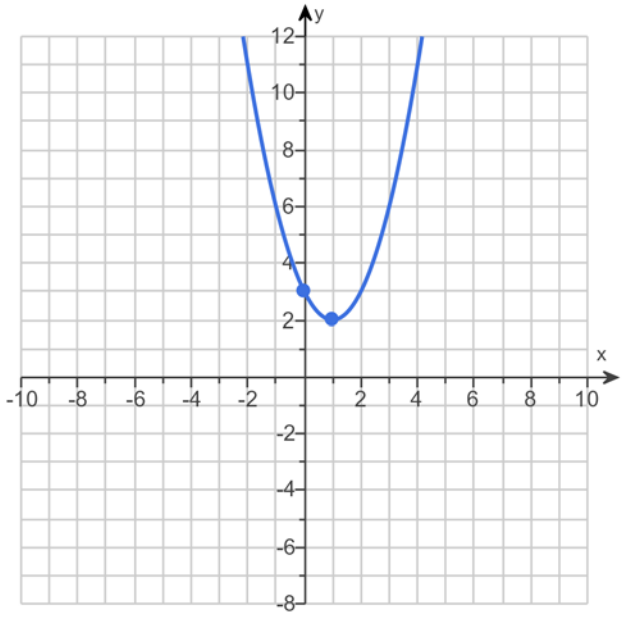


$x=-2$

$(-\infty,\infty)$

$[-9,\infty)$

20.

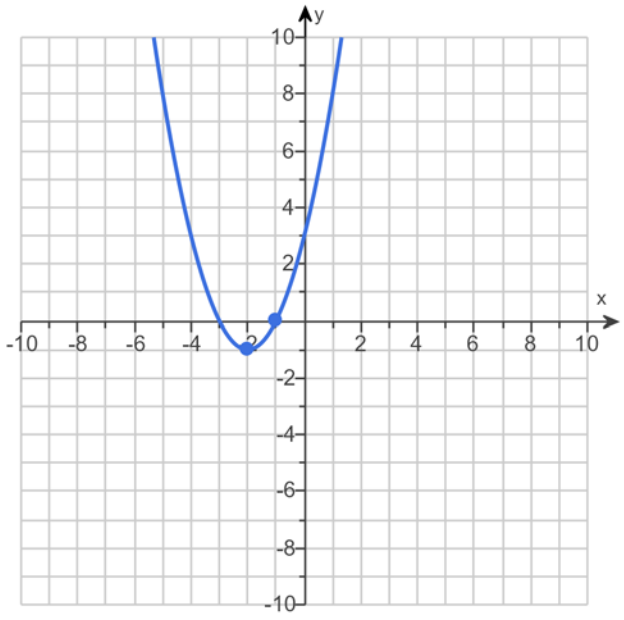


$x = 1$

$(-\infty, \infty)$

$[2, \infty)$

21.

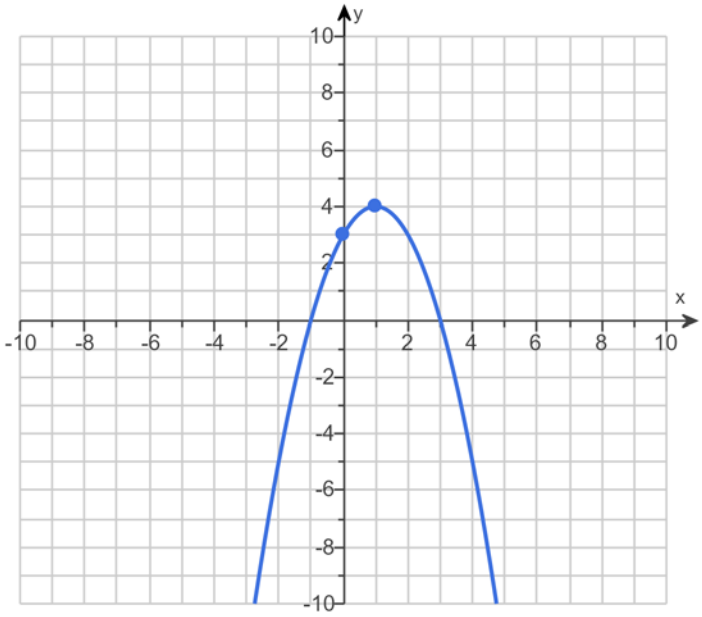


$x = -2$

$(-\infty, \infty)$

$[-1, \infty)$

22.



$x = 1$

$(-\infty, \infty)$

$(-\infty, 4]$

---

23. $2, -1, -3$

---

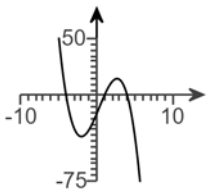
24. C. $\pm 1, \pm 3, \pm 9, \pm 5, \pm 15, \pm 45, \pm \frac{1}{7}, \pm \frac{3}{7}, \pm \frac{9}{7}, \pm \frac{5}{7}, \pm \frac{15}{7}, \pm \frac{45}{7}$

$\frac{5}{7}$

$\frac{5}{7}, 3, -3$

---

25. $-4, 1, 4$



A.

---

26. $y = 2x + 5$

---

27. B. The vertical asymptote(s) is(are) **x = 3**. There are no holes.

---

28. B. Vertical asymptote(s) at x = **5** and hole(s) at x = **6**

---

29. A. The horizontal asymptote is **y = 0**. (Type an equation.)

30. A. The horizontal asymptote is **y = 4**. (Type an equation.)

31. $(-\infty, 13)$

32. $2\log_b x + \log_b y - 3\log_b z$

33. $7\ln x + \frac{1}{2}\ln\left(x^2+4\right) - 5\ln(x+4)$

34. 19

35. $\frac{\ln 139}{5}$

0.99

36. $\frac{\ln 469}{\ln 3} - 2$

3.60

37. A. The solution set is $\{-16\}$. (Type an integer or a simplified fraction.)

38. A. The solution set is $\left\{\frac{8}{7}\right\}$. (Type an exact answer in simplified form.)

39. A. The solution set is $\{4\}$.(Simplify your answer. Use a comma to separate answers as needed.)

40. A. The solution set is $\{5\}$.(Simplify your answer. Use a comma to separate answers as needed.)

41. A. The solution set is $\{7\}$.(Simplify your answer. Use a comma to separate answers as needed.)

42. 15.7

2004

43. 10.4

44. 5

---

45. 10,230

---

46. 0.3

231

---

47. A.

There is one solution. The solution set is $\{($ **3** , **−1** , **−3** $)\}$. (Simplify your answers.)

---

48. $\frac{1}{3}$

$\frac{4}{7}$

$\frac{3}{4}$

$\frac{8}{9}$

---

49. 40

---

50. $27x^3 - 54x^2 + 36x - 8$

---

51. $x^6 + 42x^5 + 735x^4$
